

Hong Kong Mathematics Olympiad (2008 / 2009)

Final Event 1 (Group)

香港数学竞赛 (2008 / 2009)

决赛项目 1 (团体)

除非特别声明，答案须用数字表达，并化至最简。

Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

1. 已知三角形三边的长度分别是 a cm、 2 cm 及 b cm，其中 a 和 b 是整数且 $a \leq 2 \leq b$ 。若有 q 种不全等的三角形满足上述条件，求 q 的值。

Given some triangles with side lengths a cm, 2 cm and b cm, where a and b are integers and $a \leq 2 \leq b$. If there are q non-congruent classes of triangles satisfying the above conditions, find the value of q .

2. 已知方程 $|x| - \frac{4}{x} = \frac{3|x|}{x}$ 有 k 个相异实根，求 k 的值。

Given the equation $|x| - \frac{4}{x} = \frac{3|x|}{x}$ has k distinct real root(s), find the value of k .

3. 已知 x 及 y 为非零实数且满足方程 $\frac{\sqrt{x}}{\sqrt{y}} - \frac{\sqrt{y}}{\sqrt{x}} = \frac{7}{12}$ 及 $x - y = 7$ 。若 $w = x + y$ ，求 w 的值。

Given that x and y are non-zero real numbers satisfying the equation $\frac{\sqrt{x}}{\sqrt{y}} - \frac{\sqrt{y}}{\sqrt{x}} = \frac{7}{12}$ and

$x - y = 7$. If $w = x + y$, find the value of w .

4. 已知 x 及 y 为实数且 $\left|x - \frac{1}{2}\right| + \sqrt{y^2 - 1} = 0$ 。设 $p = |x| + |y|$ ，求 p 的值。

Given that x and y are real numbers and $\left|x - \frac{1}{2}\right| + \sqrt{y^2 - 1} = 0$. Let $p = |x| + |y|$, find the value of p .



Hong Kong Mathematics Olympiad (2008 / 2009)

Final Event 2 (Group)

香港数学竞赛 (2008 / 2009)

决赛项目 2 (团体)

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Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

1. 已知 $\tan \theta = \frac{5}{12}$ ，其中 $180^\circ \leq \theta \leq 270^\circ$ 。若 $A = \cos \theta + \sin \theta$ ，求 A 的值。

Given that $\tan \theta = \frac{5}{12}$, where $180^\circ \leq \theta \leq 270^\circ$. If $A = \cos \theta + \sin \theta$, find the value of A .

2. 设 $[x]$ 是不超过 x 的最大整数。若 $B = \left[10 + \sqrt{10 + \sqrt{10 + \sqrt{10 + \cdots}}} \right]$ ，求 B 的值。

Let $[x]$ be the largest integer not greater than x . If $B = \left[10 + \sqrt{10 + \sqrt{10 + \sqrt{10 + \cdots}}} \right]$, find the value of B .

3. 设 $a \oplus b = ab + 10$ 。若 $C = (1 \oplus 2) \oplus 3$ ，求 C 的值。

Let $a \oplus b = ab + 10$. If $C = (1 \oplus 2) \oplus 3$, find the value of C .

4. 在坐标平面上，用以下直线所围成图形的面积为 D 平方单位，求 D 的值。

$$L_1 : y - 2 = 0$$

$$L_2 : y + 2 = 0$$

$$L_3 : 4x + 7y - 10 = 0$$

$$L_4 : 4x + 7y + 20 = 0$$

In the coordinate plane, the area of the region bounded by the following lines is D square units, find the value of D .

$$L_1 : y - 2 = 0$$

$$L_2 : y + 2 = 0$$

$$L_3 : 4x + 7y - 10 = 0$$

$$L_4 : 4x + 7y + 20 = 0$$



Hong Kong Mathematics Olympiad (2008 / 2009)

Final Event 3 (Group)

香港数学竞赛 (2008 / 2009)

决赛项目 3 (团体)

除非特别声明，答案须用数字表达，并化至最简。

Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

1. 设 $[x]$ 是不超过 x 的最大整数。若 $A = \left\lfloor \frac{2008 \times 80 + 2009 \times 130 + 2010 \times 180}{2008 \times 15 + 2009 \times 25 + 2010 \times 35} \right\rfloor$ ，求 A 的值。

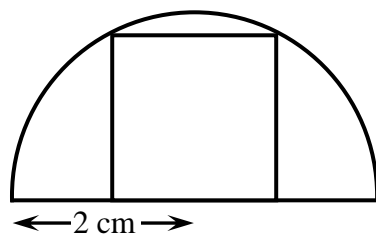
Let $[x]$ be the largest integer not greater than x . If $A = \left\lfloor \frac{2008 \times 80 + 2009 \times 130 + 2010 \times 180}{2008 \times 15 + 2009 \times 25 + 2010 \times 35} \right\rfloor$, find the value of A .

2. 在 $\underbrace{99 \dots 9}_{2009 \text{ 个 } 9} \times \underbrace{99 \dots 9}_{2009 \text{ 个 } 9} + 1 \underbrace{99 \dots 9}_{2009 \text{ 个 } 9}$ 中，末位的 0 共有 R 个，求 R 的值。

There are R zeros at the end of $\underbrace{99 \dots 9}_{2009 \text{ of } 9\text{'s}} \times \underbrace{99 \dots 9}_{2009 \text{ of } 9\text{'s}} + 1 \underbrace{99 \dots 9}_{2009 \text{ of } 9\text{'s}}$, find the value of R .

3. 如图一，边长为 Q cm 的正方形内接于半径为 2 cm 的半圆中，求 Q 的值。

In Figure 1, a square of side length Q cm is inscribed in a semi-circle of radius 2 cm. Find the value of Q .

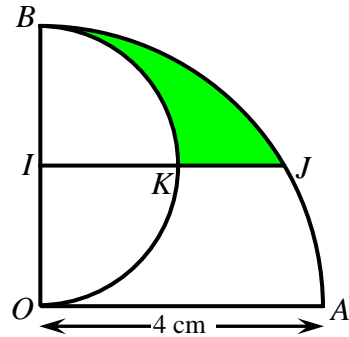


图一

Figure 1

4. 如图二，扇形 OAB 的半径为 4 cm 及 $\angle AOB$ 为直角。设以 OB 为直径的半圆，其圆心为 I 且 $IJ \parallel OA$ 及 IJ 与该半圆相交于 K 。若阴影部分的面积为 $T\text{ cm}^2$ ，求 T 的值。（设 $\pi = 3$ ）

In Figure 2, the sector OAB has radius 4 cm and $\angle AOB$ is a right angle. Let the semi-circle with diameter OB be centred at I with $IJ \parallel OA$, and IJ intersects the semi-circle at K . If the area of the shaded region is $T\text{ cm}^2$, find the value of T . (Take $\pi = 3$)



图二

Figure 2



Hong Kong Mathematics Olympiad (2008 / 2009)

Final Event 4 (Group)

香港数学竞赛 (2008 / 2009)

决赛项目 4 (团体)

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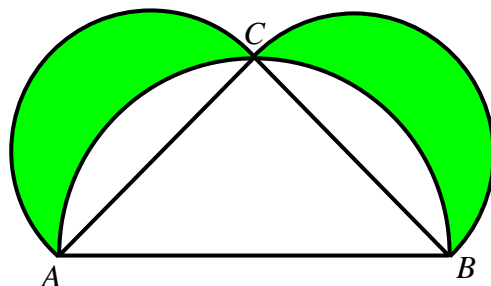
1. 设 P 为实数。若 $\sqrt{3-2P} + \sqrt{1-2P} = 2$ ，求 P 的值。

Let P be a real number. If $\sqrt{3-2P} + \sqrt{1-2P} = 2$, find the value of P .



2. 如图一，设 AB 、 AC 及 BC 为相应半圆的直径。若 $AC = BC = 1$ cm 及阴影部分的面积是 R cm²，求 R 的值。

In Figure 1, let AB , AC and BC be the diameters of the corresponding three semi-circles. If $AC = BC = 1$ cm and the area of the shaded region is R cm², find the value of R .



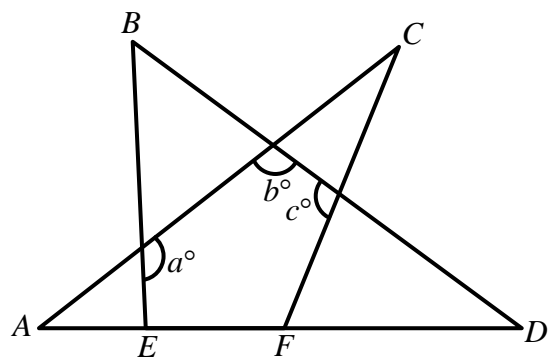
图一

Figure 1



3. 如图二， AC 、 AD 、 BD 、 BE 及 CF 为直线。若 $\angle A + \angle B + \angle C + \angle D = 140^\circ$ 及 $a + b + c = S$ ，求 S 的值。

In Figure 2, AC , AD , BD , BE and CF are straight lines. If $\angle A + \angle B + \angle C + \angle D = 140^\circ$ and $a + b + c = S$, find the value of S .



图二

Figure 2



4. 设 $Q = \log_{2+\sqrt{2^2-1}}(2-\sqrt{2^2-1})$ 。求 Q 的值。

Let $Q = \log_{2+\sqrt{2^2-1}}(2-\sqrt{2^2-1})$, find the value of Q .

